Replacing a Theoretical Probability by an Empirical Estimate

Project: Quantum clock synchronization (QCS)

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The Chain from Clock Offset to Clock Correction

The highlighted passage,

boxed clock offset □ quantum phase □ measurement probability □ statistical estimate □ clock correction

is the parent document's compact summary of the toy protocol in Section 16. It is not a proof by itself. Rather, it is a map of the reasoning: an unknown difference between Alice's and Bob's clocks is converted into a quantum phase, that phase affects the probabilities of measurement outcomes, repeated measurements allow the probability to be estimated, and the estimated probability is then inverted to infer a clock correction.

The useful way to read the chain is as an operational pipeline. A clock offset is not directly visible. Alice and Bob cannot look at an external universal time parameter. They can only prepare systems, let them evolve, measure them, and compare data. The quantum protocol works if each arrow in the chain is physically and statistically justified.

From Clock Offset to Quantum Phase

A clock offset is a difference between two time readings. If Alice's clock reads C□A□ and Bob's clock reads C□B□, the parent document defines the offset as

□ = C□B□ - C□A□

The sign convention matters. With this definition, □>0 means Bob's clock is ahead of Alice's.

Quantum mechanics enters because certain quantum states evolve periodically in time. For a two-level system with energy eigenstates |0□ and |1□, suppose the energy difference is

E□ - E□ = hbar □,

where hbar is the reduced Planck constant and □ is the angular transition frequency. If the system begins in the equal superposition

|+□=frac|0□+|1□□(2),

then, ignoring a physically irrelevant global phase, its state after time t becomes

|□(t)□ = frac|0□+e□-i□ t□|1□□(2).

The key point is that elapsed time t appears as a relative phase □ t between |0□ and |1□. Relative phase is observable through interference; global phase is not [Nielsen and Chuang 2010].

If Alice and Bob use systems with the same transition frequency □, then a clock offset □ corresponds, in the simplified model, to a phase offset

□□ = □ □.

This is the first arrow in the boxed chain:

clock offset □ quantum phase.

The parent document directly supports this equation in Sections 5 and 16. But the equation rests on assumptions that would need verification in a real protocol: Alice and Bob must agree on the relevant transition frequency, the quantum systems must remain coherent long enough for the phase to matter, and the protocol must distinguish a genuine clock offset from other phase shifts caused by the channel, apparatus, or local oscillators.

There is also an immediate ambiguity. Since quantum phase is periodic,

$$e^{i\phi} = e^{i(\phi + 2\pi)},$$

the relation $\phi = \omega\tau$ determines τ only modulo the oscillator period

$$T = \frac{2\pi}{\omega}.$$

So the phase ϕ does not by itself identify a unique clock offset unless there is additional coarse timing information, multiple frequencies, or an adaptive unwrapping procedure.

From Quantum Phase to Measurement Probability

A phase is not usually observed directly. Quantum theory predicts probabilities for measurement outcomes. The phase must therefore be converted into a measurable interference signal.

The parent document uses a Ramsey-like probability model:

$$P(0) = \frac{1 + \cos\phi}{2}.$$

Here $P(0)$ is the probability of obtaining outcome 0 in a chosen measurement basis, and ϕ is the phase caused by the clock offset. This kind of expression is characteristic of interferometric and Ramsey measurements: a phase difference changes whether probability amplitude interferes constructively or destructively [Ramsey 1950].

One simple way to see the structure is to take the phase-bearing state

$$|\psi\rangle = \frac{|0\rangle + e^{-i\phi}|1\rangle}{\sqrt{2}}$$

and measure it in the X-basis, whose "plus" state is

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}.$$

The probability of obtaining the + outcome is given by the Born rule:

$$P(+) = |\langle + | \psi\rangle|^2.$$

Computing the overlap,

$$\langle + | \psi\rangle = \frac{1 + e^{-i\phi}}{2},$$

so

$$P(+) = \left|\frac{1 + e^{-i\phi}}{2}\right|^2 = \frac{1 + \cos\phi}{2}.$$

Thus the phase becomes visible as a probability modulation. This supports the second arrow:

$$\text{quantum phase} \rightarrow \text{measurement probability}.$$

The parent document's formula is therefore plausible and standard as a toy Ramsey signal. But it is not universal. Other measurement choices can produce sine signals, shifted cosine signals, reduced-contrast fringes, or multi-outcome likelihood functions. A more realistic version might be

$$P(0 \mid □) = \frac{1+V\cos(□□+□□)}{2},$$

where V is the fringe visibility and □□ is an uncontrolled or calibrated phase offset. The toy model corresponds to the ideal case $V=1$ and $□□=0$.

This distinction matters because a real experiment estimating □ is never estimating from the bare equation alone. It is estimating through a model of the measurement apparatus.

From Measurement Probability to Statistical Estimate

A single quantum measurement does not reveal the probability. It gives one random outcome. If $P(0)=0.8535$, one trial still produces either 0 or not-0. The probability becomes experimentally accessible only through repeated trials.

Suppose Alice and Bob repeat the same experiment N times. Let $n_□$ be the number of times outcome 0 occurs. Then a natural estimate of the probability is

$$\widehat{P}(0)=\frac{n_□}{N}.$$

In the parent document's toy example,

$$N=10000,\ n_□=8535,$$

so

$$\widehat{P}(0)=0.8535.$$

This is the third arrow:

$$\text{measurement probability} □ \text{statistical estimate}.$$

The statistical model behind this step is usually binomial sampling. If every trial is independent and identically prepared, and if the true probability is $P(0)$, then

$$n_□ \sim \mathrm{Binomial}(N,P(0)).$$

The standard deviation of the estimated probability is approximately

$$□_{\widehat{P}} = □\left(\frac{P(0)(1-P(0))}{N}\right).$$

This shows why repetition improves precision. The uncertainty decreases like $1/□(N)$ for independent trials, which is the usual shot-noise scaling. Quantum-enhanced protocols may improve the scaling under particular resource assumptions, but such improvements require more careful quantum metrology analysis [Giovannetti, Lloyd, and Maccone 2006].

The parent document directly uses the simple frequency estimate 8535/10000. What is an inference beyond the parent text is the statistical justification: it assumes independent trials, stable parameters during data collection, and no systematic bias in state preparation or measurement.

From Statistical Estimate Back to Phase

The boxed chain says "statistical estimate" before "clock correction," but there is an intermediate mathematical inversion. Once Alice and Bob estimate the probability, they infer the phase.

Using the ideal formula,

$$P(0)=(1+\cos\text{[illegible]})/(2),$$

one obtains

$$\cos\text{[illegible]} = 2P(0)-1.$$

Replacing P(0) by the empirical estimate widehatP(0),