AI explanation: $e^{-i\omega t}$

Project: Quantum clock synchronization (QCS)
Document: Document 3.1.1
Author: mujirin
Verifier: Not verified

Status: Working

The Meaning of $e^{-i\omega t}$: A Qubit's Phase as a Clock Hand

The highlighted expression

$$e^{-i\omega t}$$

is a compact way of writing a rotating quantum phase. In the parent document, it appears as the time-dependent factor in the qubit state

$$|\psi(t)\rangle = \frac{|0\rangle + e^{-i\omega t}|1\rangle}{\sqrt{2}}.$$

The parent document directly supports the interpretation that this factor is what makes the qubit "tick": not by producing a physical click, but by changing the relative phase between the two components of a superposition at a predictable angular frequency ω. The expression itself is small, but it carries the central physics of a two-level quantum clock.

A Complex Number Rotating in Time

The quantity $e^{-i\omega t}$ is a complex number of unit magnitude. By Euler's formula,

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t).$$

Here i is the imaginary unit, t is time, and ω is an angular frequency, measured in radians per second. As t increases, the point $e^{-i\omega t}$ moves around the unit circle in the complex plane. Its magnitude remains fixed:

$$\left|e^{-i\omega t}\right| = 1,$$

so it does not make the $|1\rangle$ part of the qubit larger or smaller. It only changes its phase.

This distinction matters. A coefficient such as $2|1\rangle$ would change an amplitude's size relative to other components. By contrast, multiplying by $e^{-i\omega t}$ changes only the direction of the complex coefficient. In quantum mechanics, such phase directions can be physically meaningful when they are relative phases between different components of a superposition.

In the parent document's state,

$$|\psi(t)\rangle = \frac{|0\rangle + e^{-i\omega t}|1\rangle}{\sqrt{2}},$$

the two components still have equal magnitude. The probabilities of finding $|0\rangle$ or $|1\rangle$ in a direct energy-basis measurement remain 1/2 and 1/2. What changes is the relative phase between them. That changing relative phase is the "clock hand."

Where the Factor Comes From

The standard source of $e^{-i\omega t}$ is the Schrödinger time-evolution rule. If a quantum state has definite energy E, then under a time-independent Hamiltonian it evolves as

$$|\psi(t)\rangle = e^{-iEt/\hbar}|\psi(0)\rangle,$$

where $\hbar$ is the reduced Planck constant. This is one of the basic consequences of the time-dependent Schrödinger equation [Sakurai and Napolitano 2017].

For a two-level system whose basis states $|0\rangle$ and $|1\rangle$ are energy eigenstates, suppose

H|0□=E□|0□, H|1□=E□|1□.

If the initial state is

|□(0)□=frac|0□+|1□□(2),

then each energy component accumulates its own phase:

|□(t)□ = frace□-iE□t/hbar□|0□+e□-iE□t/hbar□|1□□(2).

The overall factor e□-iE□t/hbar□ multiplies the entire state, so it is a global phase. Global phases are not directly observable in ordinary quantum measurements. Factoring it out gives

|□(t)□ = e□-iE□t/hbar□ frac|0□+e□-i(E□-E□)t/hbar□|1□□(2).

Dropping the physically irrelevant global phase leaves

|□(t)□ = frac|0□+e□-i□ t□|1□□(2),

with

□=(E□-E□)/(hbar).

Thus, in the parent document's interpretation, □ is not an arbitrary symbol. It is the angular frequency associated with the energy splitting between the two qubit states. This is the connection that lets a qubit behave like a clock: energy difference becomes phase evolution, and phase evolution becomes a time marker.

Why the Phase Is Relative

A crucial point is that e□-i□ t□ is meaningful here because it is attached to only one branch of the superposition relative to the other. If the whole state were multiplied by the same factor,

|□(t)□ □ e□-i□ t□|□(t)□,

then the factor would be a global phase. Such a phase does not change measurement probabilities and is normally unobservable.

But in

frac|0□+e□-i□ t□|1□□(2),

the phase changes the relationship between |0□ and |1□. That relative relationship can be revealed by interference measurements. For example, measuring only in the \|0□,|1□\ basis cannot see the phase, because both outcomes still have equal probability. But measuring in a superposition basis can convert phase into population differences. This is the same broad logic behind Ramsey interferometry, where accumulated phase is inferred through later interference rather than directly observed as a visible hand on a dial [Ramsey 1950].

So the expression e□-i□ t□ should not be read as "the qubit visibly displays time." It means that, if the qubit remains coherent and if one has an appropriate phase reference, the relative phase evolves predictably with time.

The Sign in the Exponent

The minus sign in

e□-i□ t□

comes from the conventional form of Schrödinger evolution,

$$U(t)=e^{-iHt/\hbar}$$

With the usual convention that $E_1>E_0$, the relative phase of $|1\rangle$ compared with $|0\rangle$ appears as

$$e^{-i(E_1-E_0)t/\hbar}$$

One should be careful not to overinterpret the sign by itself. Some authors define the Hamiltonian, basis states, rotating frame, or phase convention differently. In another convention one might see $e^{+i\omega t}$. The physical content is not the isolated sign but the predicted relative phase and the measurement statistics that follow from it. To verify the parent document's expression, the reader should check which state has the higher energy and which global phase has been factored away.

The Periodic Nature of the Tick

Because the phase lies on the unit circle, it repeats whenever

$$\omega t \mapsto \omega t+2\pi$$

The phase therefore has period

$$T=\frac{2\pi}{\omega}.$$

This means the qubit's phase does not encode unlimited absolute time by itself. It encodes time modulo one period unless additional information is supplied. If $\omega t=0, 2\pi$, or 4π, the phase factor is again 1. If $\omega t=\pi$, the phase factor is -1, and the state becomes

$$\frac{|0\rangle-|1\rangle}{\sqrt{2}}.$$

On the Bloch sphere, this corresponds to an equatorial state rotating around the vertical axis. The "tick" is continuous rotation, not a sequence of discrete clicks. The period $2\pi/\omega$ is the time for the phase hand to return to its original direction.

What Must Be True for This Expression to Describe a Clock

The parent document's use of $e^{-i\omega t}$ is theoretically standard, but its application to transported "ticking qubits" depends on several assumptions. The two basis states should behave like stable energy eigenstates, or at least like states whose relative phase evolves in a known way. The energy splitting should be known well enough that ω can serve as a clock frequency. The superposition must remain coherent during transport, because decoherence would destroy the off-diagonal phase information that $e^{-i\omega t}$ represents. Finally, Alice and Bob need a way to compare the qubit phase against local phase references; without such a reference, the phase is not an absolute timestamp.

Thus the highlighted factor is best understood as the mathematical trace of the qubit's internal phase evolution. It is directly supported by the parent document as the term that "ticks," but verifying the reasoning requires checking the Hamiltonian convention, the removal of global phase, the relation $\omega=(E_1-E_0)/\hbar$, and the operational procedure by which the relative phase is eventually measured.

References

[Ramsey 1950] N. F. Ramsey, "A Molecular Beam Resonance Method with Separated Oscillating Fields," Physical Review 78, 695–699 (1950). DOI: 10.1103/PhysRev.78.695

[Sakurai and Napolitano 2017] J. J. Sakurai and J. Napolitano, Modern Quantum Mechanics, 2nd ed., Cambridge University Press, 2017.

[Nielsen and Chuang 2010] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, 10th Anniversary ed., Cambridge University Press, 2010.