Quantum Eddington Slow-Clock Transport

Project: Quantum clock synchronization (QCS)

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The phrase Quantum Eddington Slow-Clock Transport sounds unusual, but its meaning becomes clear if we separate it into two layers. The first layer is Eddington slow-clock transport, a classical synchronization method. The second layer is its quantum analogue, where the transported clock is no longer an ordinary wristwatch but a quantum system, usually a two-level system whose phase evolves in time. In recent quantum-clock-synchronization literature, this quantum version is often described using the language of ticking qubits.

Classically, the clock-synchronization problem is simple to state. Alice and Bob are far apart. Alice has a clock, Bob has a clock, and both clocks tick at the same rate, but they may not agree on the zero of time. The unknown quantity is the offset

$$\delta = t_B - t_A$$

If $\delta=0$, the clocks are synchronized. If $\delta=10$ ns, Bob's clock is ten nanoseconds ahead of Alice's. The difficulty is that Alice and Bob cannot simply put their clocks side by side after they have already separated. Any comparison over distance seems to require either sending a signal or physically transporting a clock.

Einstein synchronization uses the first method. Alice sends a light pulse to Bob, Bob reflects it, and Alice assumes that the outward and return travel times are equal. Eddington slow-clock transport uses the second method. Alice first synchronizes a portable clock with her own clock. Then she transports that clock slowly to Bob. When the clock arrives, Bob compares it with his local clock. This avoids the central weakness of signal-based synchronization: the need to know the message delivery time. De Burgh and Bartlett describe this Eddington protocol exactly in this spirit: Alice synchronizes a portable "wristwatch" with her own clock, transports it adiabatically to Bob, and Bob compares it with his clock to estimate the time difference.

The word slow is not decorative. It is there because a moving clock experiences time dilation. In special relativity, if the transported clock moves at speed v, then its proper time satisfies approximately

$$d\tau \approx dt\left(1-\frac{v^2}{2c^2}\right),$$

when $v \ll c$. The moving clock ticks slightly less time than a clock left at rest in the same inertial frame. If the transport is made slower and slower, the correction becomes smaller and smaller. That is the classical reason Eddington's method asks for slow transport.

For example, suppose Alice and Bob are separated by $L=1$ km, and Alice carries a clock to Bob at $v=1$ m/s. The travel time is

$$T=\frac{L}{v}=1000 \text{ s}.$$

The special-relativistic time-dilation error is approximately

$$\Delta T \approx \frac{v^2}{2c^2}T = \frac{vL}{2c^2}.$$

Using $c \approx 3\times 10^8$ m/s, we get

$$\Delta T \approx \frac{1\times 1000}{2(3\times 10^8)^2} \text{ s} \approx 5.6\times 10^{-15} \text{ s}.$$

That is only a few femtoseconds. In the ideal mathematical limit $v\to 0$, the special-relativistic transport error tends to zero. In real experiments, however, one cannot move infinitely slowly, because clocks drift, environments change, and gravity may alter the clock rate.

The quantum version begins by asking a beautiful question: what is the smallest possible portable clock? A very natural answer is: a two-level quantum system. Let the two energy states be $|0\rangle$ and $|1\rangle$, and suppose their energy difference corresponds to angular frequency ω. A quantum clock can be prepared in the superposition

$$|+\rangle=\frac{|0\rangle+|1\rangle}{\sqrt{2}}.$$

Under its own Hamiltonian, this state evolves into

|□(t)□ = frac|0□+e□-i□ t□|1□□□(2).

The phase □ t is the ticking of the qubit. It is the quantum analogue of the hand of a clock.

Now we can understand quantum Eddington slow-clock transport. Alice prepares a ticking qubit using her local clock as the phase reference. She then transports or sends that qubit to Bob in a way that preserves its coherence. Bob measures the qubit relative to his own clock. If Bob's clock is offset by □, the qubit's phase appears shifted by

□□ = □ □.

So Bob's task is no longer to read a mechanical hand. His task is to estimate a quantum phase. Once he estimates □□, he obtains

□ = (□□)/(□).

For example, suppose Alice and Bob use a transition frequency

f=100 MHz,

so that

□=2□ f=2□□ 10□ rad/s.

If Bob's clock is ahead by

□=1 ns,

then the phase shift is

□□ =□□ =2□□ 10□□ 10□□ =0.2□.

A one-nanosecond clock offset has therefore become a phase shift of 0.2□ radians.

But Bob cannot simply look at one qubit and read off the phase. A quantum measurement is probabilistic. If Bob measures in the right interference basis, he may obtain a probability such as

P(+)=(1+cos(□□))/(2).

This equation is the operational heart of the protocol. The clock offset □ is invisible directly, but it changes a measurable probability.

Using the same example, □□=0.2□, so

P(+)=(1+cos(0.2□))/(2)□0.9045.

If Bob receives many identically prepared ticking qubits and obtains the + outcome in about 905 out of 1000 trials, he estimates

$$P(+) \approx 0.905.$$

Then

$$\cos(\omega\Delta t) \approx 2(0.905)-1=0.81,$$

so

$$\omega\Delta t \approx \arccos(0.81) \approx 0.626 \text{ rad}.$$

Finally,

$$\Delta t \approx \frac{0.626}{2\pi \times 10^9} \text{ s} \approx 1.0 \text{ ns}.$$

This shows the whole mechanism in one chain:

$$\text{clock offset} \longrightarrow \text{qubit phase} \longrightarrow \text{measurement probability} \longrightarrow \text{statistical estimate of time offset}.$$

The quantum idea is therefore not that the qubit magically tells Bob the time. Rather, the qubit carries a coherent phase reference from Alice to Bob. Bob compares that phase reference with his own clock. The comparison is done by interference, and the result is extracted statistically.

This is why the qubit is often called a ticking qubit. It is not ticking by moving a pointer around a dial. It is ticking because the relative phase between $|0\rangle$ and $|1\rangle$ rotates at a known angular frequency ω. De Burgh and Bartlett describe quantum Eddington-type protocols as clock-synchronization methods using the adiabatic exchange of nondegenerate two-level quantum systems, or ticking qubits. Their paper also notes that independent ticking-qubit exchanges achieve the standard quantum limit scaling

$$\Delta t \sim \frac{1}{\omega\sqrt{N}},$$

where N is the number of independent qubits.

This scaling has a simple undergraduate meaning. If one qubit gives a noisy estimate of the phase, then many independent qubits allow averaging. Ordinary statistical averaging improves like $1/\sqrt{N}$. Therefore, if $N=10{,}000$, the uncertainty is roughly 100 times smaller than the uncertainty from one qubit.

For example, with

$$\omega=2\pi \times 1 \text{ GHz}$$

and

$$N=10^4,$$

the rough standard-quantum-limit time uncertainty is

$$\Delta t \sim \frac{1}{\omega\sqrt{N}} = \frac{1}{2\pi \times 10^9 \times 100} \approx 1.6 \text{ ps}.$$

So higher ticking frequency and more repeated quantum probes both improve synchronization accuracy.

The more surprising part is that quantum mechanics may do better than independent averaging. Chuang's distributed quantum clock-synchronization algorithm showed that, under a model with uncertain message delivery times, a quantum algorithm can obtain n digits of the clock offset using only O(n) quantum messages, whereas the corresponding classical requirement can scale as O(2□□) messages. De Burgh and Bartlett later analyzed ticking-qubit protocols and described a coherent-exchange protocol that beats the standard quantum limit without using entanglement, achieving a scaling of approximately

$$\square t \sim (\log N_{\square c \square})/(\square N_{\square c \square}),$$

where $N_{\square c \square}$ counts coherent exchanges of a single qubit.

The phrase coherent exchange is important. It means that the qubit is not measured and destroyed at every step. Instead, it is sent back and forth while preserving its phase coherence. Each coherent pass accumulates more phase information. In effect, repeated coherent transport can make the qubit behave as if it had ticked more strongly, somewhat like increasing the effective interrogation time in interferometry.

For example, suppose a single pass gives phase □□. If the protocol coherently arranges for the relevant phase to accumulate m times, then the measured phase can behave like

$$m\square\square.$$

A small time offset then produces a larger measurable phase. This is useful because small phases are hard to distinguish from zero, while amplified phases are easier to estimate. The price is that coherence must be preserved throughout the multiple exchanges. Loss, decoherence, uncontrolled phase shifts, and imperfect operations can destroy the advantage.

This also clarifies the difference between quantum Eddington synchronization and entanglement-based synchronization. Entanglement-based protocols use shared nonclassical correlations between Alice and Bob. Quantum Eddington protocols, in their simplest form, need not use entanglement. They can use a single transported quantum clock. The quantum resource is not necessarily entanglement; it can be coherence, phase evolution, and controlled quantum transport.

There is one more subtle point. In classical Eddington transport, a wristwatch can be read many times. In quantum Eddington transport, a single qubit generally cannot reveal its phase perfectly, because measurement gives only one random outcome and usually disturbs the state. Therefore, quantum clock transport naturally becomes a problem in quantum estimation theory. Alice and Bob must decide what states to send, what measurements to perform, how many repetitions to use, and how to handle phase ambiguity.

Phase ambiguity appears because

$$e^{\square i \square\square} = e^{\square i(\square + 2\square)\square}.$$

If Bob estimates □□=0.2□, this could mean 0.2□, or 2.2□, or 4.2□, and so on. In time units, the ambiguity period is

$$T = (2\square)/(\square) = (1)/(f).$$

If f=100 MHz, then

$$T = 10 \text{ ns}.$$

So a phase estimate corresponding to 1 ns could also correspond to 11 ns, 21 ns, and so forth. A practical protocol therefore needs either a coarse prior estimate, several frequencies, or an adaptive procedure that first finds the large-scale offset and then refines it.

The practical limitations are as important as the ideal formula. The transported qubit must remain coherent. The energy splitting □ must be stable. The quantum channel must not introduce uncontrolled phase noise. Bob's measurement apparatus must be calibrated. If a physical clock is transported through different gravitational potentials, relativistic frequency shifts must be corrected. Thus quantum Eddington slow-clock transport is not a way to escape relativity or engineering noise. It is a way to reformulate clock transport as quantum phase transport.

The cleanest summary is this: classical Eddington slow-clock transport carries a synchronized clock from Alice to Bob; quantum Eddington slow-clock transport carries a synchronized quantum phase from Alice to Bob. In the classical case, Bob reads a wristwatch. In the quantum case, Bob estimates the phase of a ticking qubit. The fundamental equation is

$$\square\square = \square\square$$

Everything else is a matter of how well Alice and Bob can prepare, preserve, transport, interfere, and measure that phase.

References

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