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Project: Scrodinger Equation

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Author: mujirin

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The Schr□dinger equation is the fundamental governing equation of non-relativistic quantum mechanics. It plays the same role in the quantum world that Newton's second law (F = ma) plays in classical mechanics. Instead of predicting a precise trajectory, it calculates the wavefunction, which contains all the statistical information about a system.

Here is the mathematical breakdown of its two main forms.

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1. The Time-Dependent Schr□dinger Equation (TDSE)

This is the most general form of the equation. It describes how a quantum system evolves over time.

ihbar (□)/(□ t) □(r, t) = hatH □(r, t)

Breaking Down the Components:

i: The imaginary unit (□(-1)). Its presence ensures that quantum states wave and oscillate rather than simply decaying like heat.

hbar: The reduced Planck constant (hbar = (h)/(2□)), which sets the scale of the quantum world.

(□)/(□ t): The partial derivative with respect to time, representing how the system changes second by second.

□(r, t): The wavefunction. It is a function of position r and time t. The absolute square |□|□ gives the probability density of finding a particle at a specific place and time.

hatH: The Hamiltonian operator. In quantum mechanics, physical observables are represented by operators. The Hamiltonian represents the total energy of the system.

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2. The Hamiltonian Operator in Detail

To do actual calculations for a particle moving in space, we must expand the Hamiltonian operator hatH into its kinetic and potential energy components:

hatH = hatT + hatV

Where:

Kinetic Energy Operator (hatT): Derived from momentum, it is expressed as -(hbar□)/(2m)□□, where m is the mass and □□ (del-squared) is the Laplacian operator checking spatial curvature.

Potential Energy Operator (hatV): Represented by V(r, t), which depends on the environment (e.g., an electric field or a gravitational well).

Putting it all together in 3D space:

ihbar (□)/(□ t) □(r, t) = □ft[ -(hbar□)/(2m)□□ + V(r, t) ] □(r, t)

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3. The Time-Independent Schr□dinger Equation (TISE)

If the potential energy V does not depend on time (V(r, t) = V(r)), we can use a mathematical technique called separation of variables. We split the wavefunction into a spatial part and a temporal part: □(r, t) = □(r) e□-iEt/hbar□

Plugging this back in yields the Time-Independent Schr□dinger Equation:

hatH□(r) = E□(r)

Or explicitly:

□ft[ -(hbar□)/(2m)□□ + V(r) ] □(r) = E□(r)

Mathematical Significance of the TISE:

An Eigenvalue Problem: Mathematically, this is an eigenvalue equation. hatH is a linear operator, □(r) is the eigenfunction, and E is the eigenvalue.

Quantization: Solving this equation under specific boundary conditions restricts the allowable values of E. This is precisely where "quantization" comes from in quantum mechanics□energy levels cannot be just anything; they must be specific discrete values.

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Summary of Mathematical Interpretations

| Concept | Mathematical Representation | Meaning | | --- | --- | --- | | Probability Conservation | □□-□□□□□ | □(r, t) | | Linearity | If\psi1and\psi2are solutions, thenc1\psi1 + c2\psi2is also a solution. | Explains quantum superposition. | | Hermitian Operator |\hat{H} = \hat{H}^\dagger| Guarantees that the energy eigenvalues (E$) will always be real numbers, which can be measured in a lab. |