

wavefunction

|                                     |                        |
|-------------------------------------|------------------------|
| Project: Schrodinger Equation       | Document: Document 1.1 |
| Author: mujirin                     | Verifier: Not verified |
| Downloaded: June 14, 2026 05:24 KST | Status: Working        |

In quantum mechanics, a wavefunction is a mathematical description of the quantum state of an isolation system. It contains all the physical information available about that system.

While classical mechanics tracks a particle's exact position and momentum over time using vectors, quantum mechanics replaces that predictability with a wave of probabilities.

Here is the concept broken down mathematically and conceptually.

---

1. The Mathematical Definition

The wavefunction is usually denoted by the Greek letter Psi:  $\Psi(r, t)$  for a time-dependent state, or  $\psi(r)$  for a time-independent state, where  $r$  is the position vector and  $t$  is time.

Mathematically, it is a complex-valued function. This means that at any point in space, the value of the wavefunction is a complex number (e.g.,  $a + bi$ ). Because it is complex, a raw wavefunction cannot be directly measured in a lab.

---

2. The Born Interpretation: Probability Density

In 1926, physicist Max Born provided the crucial link between the abstract mathematics of the wavefunction and the real world. He stated that while  $\Psi$  itself isn't directly observable, the square of its absolute value represents a probability density.

$$P(r, t) = |\Psi(r, t)|^2 = \Psi^* \times \Psi(r, t)$$

Where  $\Psi^*$  is the complex conjugate of  $\Psi$

If  $|\Psi(r, t)|^2$  is high at a certain point, there is a high probability of finding the particle there.

If it is zero, the particle will never be found there.

---

3. Mathematical Constraints (The Rules of a Wavefunction)

For a wavefunction to represent a physically real particle, it must satisfy a strict set of mathematical boundary conditions:

A. Normalization

The particle must exist somewhere in the universe. Therefore, if you integrate the probability density over all space, the total probability must equal 1 (or 100%).

$$\int_{-\infty}^{\infty} |\Psi(r, t)|^2 d^3r = 1$$

If a function cannot be normalized (the integral equals infinity), it cannot represent a real, localized physical particle.

B. Continuity and Single-Valuedness

To be a valid solution to the Schrodinger equation,  $\Psi(r)$  must be:

- 1. Single-valued: A particle cannot have two different probabilities of being in the exact same spot.
- 2. Continuous: There cannot be abrupt "gaps" or steps in space where the wavefunction suddenly vanishes.
- 3. Continuously differentiable: Its first spatial derivative ( $\partial\Psi/\partial x$ ) must also be continuous (except where the potential energy  $V$  becomes infinite). This ensures the particle's momentum is well-defined.

---

#### 4. Extracting Real Information: Operators and Expectation Values

Since you can't measure  $\psi$  directly, how do you find things like a particle's position, momentum, or energy? You use operators.

In quantum mechanics, every physical observable is associated with a linear, Hermitian operator. When you want to find the average outcome of many identical measurements (called the expectation value), you sandwich the operator between the wavefunction and its conjugate:

$$\langle A \rangle = \int \psi^* \hat{A} \psi \, d\mathbf{r}$$

Where  $\hat{A}$  is the operator for the property you want to measure. For example:

Position operator ( $\hat{x}$ ): Just multiplies the wavefunction by  $x$ .

Momentum operator ( $\hat{p}$ ): Involves taking a spatial derivative:  $-\hbar \nabla$  ( $\nabla / i$ ).

---

Summary: What does it actually mean?

Think of the wavefunction as a "cloud of possibilities." Until you interact with the particle (make a measurement), the particle doesn't exist at one specific point; it exists as a superposition of all possible points defined by  $\psi$ .

The moment a measurement is made, the wavefunction is said to "collapse" into a single, sharp spike, giving you a definitive, classical reading for position or momentum.